

Spur, Helical, and Spiral Bevel Transmission Life Modeling

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Computer estimation of the life, dynamic capacity, and reliability of aircraft transmissions enables comparisons of transmission service life at the design stage for optimization. Through modular programming, a variety of transmissions may be analyzed with the same software, including spur, helical, and spiral bevel reductions as well as series combinations of these reductions. The basic spur and helical reductions include, single mesh, compound, and parallel path plus reverted star and planetary gear trains. The spiral bevel reductions include single and dual input drives with arbitrary shaft angles. The analysis may be performed in either the SI metric or the English inch system of units. The reliability and life analysis are based on the two-parameter Weibull distribution lives of the component gears and bearings. A program output file describes the overall transmission and each constituent transmission, its components, and their locations, capacities, and loads. Primary output is the dynamic capacity and 90% reliability and mean lives of the unit transmissions and the overall system that can be used to estimate service overhaul frequency requirements. Two examples are presented to illustrate the information available for single element and series transmissions.

Nomenclature

a = life adjustment factor
 C = dynamic capacity, N
 F = load, N
 l = life, million cycles or hours
 R = reliability
 v = load adjustment factor

Subscripts

a = adjusted
 av = average
 i = counter
 n = total number
 r = radial
 s = system
10 = 90% reliability

Superscripts

b = Weibull slope
 p = load-life factor

Introduction

NEW transmissions that are lighter, smaller, and longer lasting than present transmissions would improve aircraft performance.¹ The life of an aircraft transmission in this context is really the service time between overhauls,² which is related to a transmission's eventual fatigue life in an unserved state. Since testing for the eventual fatigue lives of pro-

TOTYPE transmissions is very time consuming and expensive, optimization by experimentation is not reasonable.

However, computer optimization at the design stage offers the promise of improved transmission capability with affordable development costs.^{3,4} Computer programs are available for the life analysis of different bearings and bearing shaft configurations using the Lundberg–Palmgren fatigue life model. This theory has also been applied to the analysis of fatigue lives of spur and helical gears based on surface pitting as the eventual mode of failure.⁵ The statistical basis of the reliability analysis is the two-parameter Weibull distribution.

The theory has been verified with a comparison analysis of the service life requirements for an Allison T56/501 turboprop reduction gearbox.⁶

There are also analysis programs for life and dynamic capacity at a given reliability for planetary and bevel gear transmissions^{7–9} and for a number of parallel shaft reductions.¹⁰

These simulations are limited in the number of configurations they can analyze. However, the parallel shaft reduction program (Pshaft)¹⁰ was written in a modular form to allow the addition of new configurations. The current simulation, which is now called Tlife, adds significantly to the number and complexity of transmissions that can be analyzed. Helical gear transmissions and single and dual input spiral bevel transmissions have been added to the basic transmission set and a method of combining basic transmissions to simulate more complex systems has been devised. This saves the effort and time involved in developing code to simulate a new transmission that may be composed of the same unit transmissions in a different order. Also, the modular construction of the program simplifies adding a new unit configuration to the program.

The life and reliability analysis in this simulation is based on the assumption that the transmissions are well lubricated and the gears well designed and of high quality so that failure by tooth breakage and tip scoring are avoided and surface pitting is the only mode of failure.^{2,5,6}

An input file includes the transmission configuration, specification of input torque, and input speed for the first transmission and the locations, types, and basic dynamic capacities

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of the gears and bearings for all unit transmissions of which the overall transmission is composed from input to output. The basic dynamic capacities are the forces that the components can carry for one-million load cycles with a 90% probability of survival.

For each unit transmission analyzed, the output file includes an echo of the input data; a listing of the component loads, basic dynamic capacities, life factors, geometries, and adjusted dynamic capacities; followed by a listing of each component's and the unit transmission's output dynamic capacity, load-life, and Weibull exponents, and 90% and mean lives. Here the adjusted capacities are the basic dynamic capacities of the components in units of force including the life and load adjustment factors and the output dynamic capacities are the output torques required to cause loads of these magnitudes on the components.

Examples of a spiral bevel gear design and a compound helical gear design are presented to illustrate the flexibility of the analysis. The compound helical gear example can be analyzed directly as a compound transmission or as two single mesh reductions in series. Preceding the examples, short descriptions of the program structure and analyses methods are given.

The program enables a computer to estimate the service life, dynamic capacity, and reliability of significantly different transmissions with the same measure. Instead of focusing on a specific geometry, the analysis provides a tool for comparing alternative designs objectively before the expensive prototype stage.

Analysis Capability

The program can analyze 11 basic transmission configurations, which are composed of spur, helical, and spiral bevel gears. These unit reductions, listed in the order of program analysis, are 1) the single mesh spur gear reduction, 2) the single mesh helical gear reduction, 3) the compound spur gear reduction, 4) the compound helical gear reduction, 5) the parallel compound spur gear reduction, 6) the parallel compound helical gear reduction, 7) the reverted spur gear planetary or star reduction, 8) the reverted helical gear planetary or star reduction, 9) the single plane spur gear planetary or star reduction, 10) the spiral bevel reduction, and 11) the dual spiral bevel reduction.

Figure 1 is a schematic of the spur and helical single-mesh reductions in this list, which are items 1 and 2. In the figure, both an external final gear and an internal final gear are shown. Figure 2 is a schematic of the spiral bevel single mesh reductions, which are items 10 and 11. The dual spiral bevel reduc-

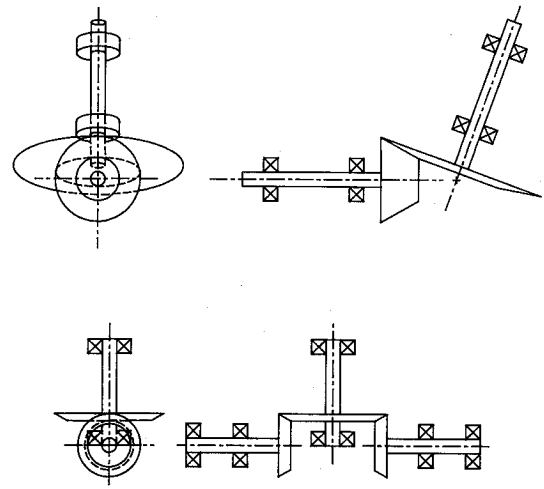


Fig. 2 Single mesh spiral bevel unit transmissions: a) single and b) dual output.

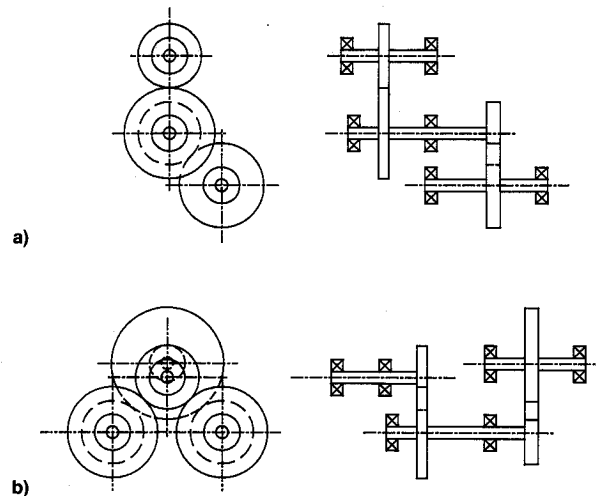


Fig. 3 Compound mesh unit transmissions: a) compound and b) parallel compound spurs and helical.

tion is a single reduction with two input gears and a single combining output gear. Figure 3 is a schematic of the compound mesh reductions in the list, which are items 3–6; and Fig. 4 is a schematic of the planetary and star unit reductions that comprise items 7–9 in the list.

Each single mesh reduction has single gears mounted on its input and output shafts supported by two bearings each. These gears can be supported either in straddle or in overhung configurations. The bearings can be single- or double-row ball or cylindrical roller or double-row tapered roller. For the helical and spiral bevel gear cases, at least one of the bearings on each shaft must be either a ball bearing or a tapered roller bearing to support the axial load. For the dual spiral bevel reduction, the two input pinions are assumed to be identical in all respects.

The compound mesh reductions, shown in Fig. 3, have input and output shafts that are similar to those of the single mesh reductions. They also have one or two intermediate shafts that support two intermediate gears on two bearings. The input and intermediate gears are external gears and the output gear may be an external gear or an internal ring gear. The intermediate gears may be supported in one of four ways: 1) double straddle, 2) double overhung, 3) output gear overhung, or 4) input gear overhung. For the parallel compound spur and helical reductions it is assumed that the two intermediate shafts are identical in all respects. The output shaft may be on the same

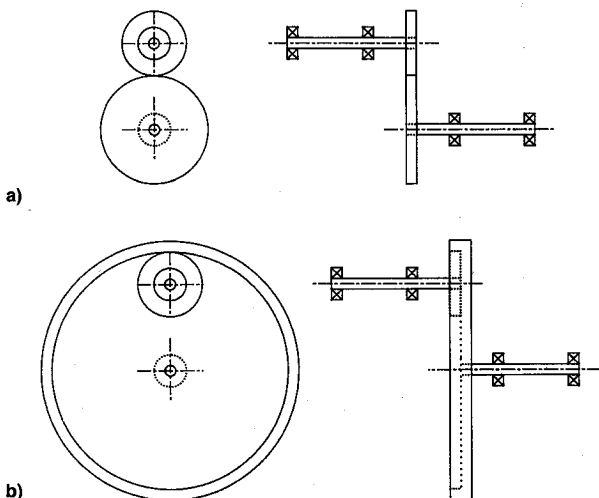


Fig. 1 Single mesh spur and helical unit transmissions: a) external and b) internal output gear.

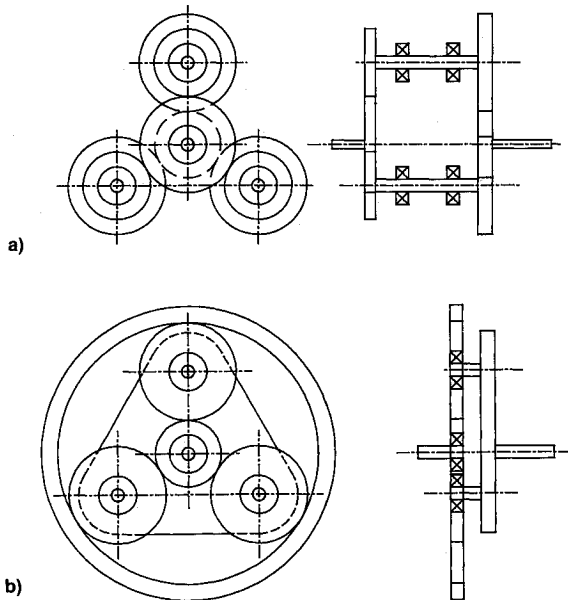


Fig. 4 Planetary and star unit transmissions: a) reverted spur and helical and b) single plane spur.

side of the intermediate shafts as the input shaft shown in Fig. 3 or it may be on the opposite side.

The planetary and star reductions, which are shown schematically in Fig. 4, include the reverted reductions and the single plane reduction that can only be a spur reduction since the program's analysis assumes there are no axial loads on the single plane planet bearings. The reverted reductions have an input gear, a final gear, which may be external or internal, and at least two identical intermediate shaft assemblies with two gears and two bearings symmetrically spaced about the collinear input and output gears. The output of the reduction unit may be either the final gear, with the arm holding the intermediate shaft subassemblies fixed, or the arm with the final gear fixed. Taking the output from the arm makes this configuration work as a reverted planetary reduction.

The single plane reduction has an input sun gear and a final ring gear in mesh with a number of planets that are symmetrically placed about the sun and ring gears. This makes the input and output shafts collinear. The planet gears may be unstepped or stepped, with one size of gears meshing with the sun gear and another size of concentric gears meshing with the ring gear. In order to keep the loads on the planet bearings radial for a transmission with stepped planets, one set of planet gears must be split and straddle the other. The radial loads coming on the planet gears are carried by single, in-plane bearings, fixed relative to the inner race. The output of the reduction unit may be either the final gear, with the arm holding the planets fixed, or the arm with the ring gear fixed for a planetary reduction.

In addition to single unit reductions, a series of reduction units may be combined for analysis or a two-branch reduction may be analyzed with the dual spiral bevel unit acting as a combining element. The dual spiral bevel need not be the first unit in the reduction, but its dual inputs are assumed to be equal, so the two branches that feed it must be identical.

Program Use

Because of the large amount of input data required to define a reduction, the transmission to be analyzed is described in an input data file before running the program. A detailed description of the input file and use of the program Tlife are available.¹¹

In series transmissions in which a unit transmission's output shaft and the following unit transmission's input shaft share bearings, two bearings are specified for the output shaft of the

driving unit and two bearings are specified for the input shaft of the driven unit. Physically, this is the same shaft that only has two bearings. In the analysis, the bearings on the output shaft of the driving unit are used only to determine loads. These loads are superimposed on the bearings of the input shaft of the driven unit along with the input shaft's own loads. The life and reliability of the input shaft bearings are then calculated for the total loads and used in the system life analysis.

For each unit transmission an output file gives a description of the transmission configuration, transmission characteristics, and the input data for the gears and bearings, followed by a report of the loads and basic dynamic capacities. Following this is a summary of the output dynamic capacities in units of output torque, the 90% reliability lives in million output rotations and hours, and the mean lives in hours for the different components and the unit transmission. The output dynamic capacity and life summary is given for the overall transmission with the overall mean life given as a transmission life and an overall component life. The transmission life predicts the mean life between service overhauls with full transmission replacement, whereas the overall component mean life predicts only the mean time between service overhauls for maintenance by a failed component replacement.

For transmissions composed of a series of unit transmissions, the output file starts with the overall reduction input and output speeds, torques and rotational directions along with the overall speed reduction, and the transmitted power. Then separate unit transmission analyses as described previously are given for each unit transmission. Finally, the dynamic capacity and lives of the overall reduction are written to the end of the output file.

Program Structure

The structure of the computer program that performs these analyses is shown in the block diagram of Fig. 5. A main program opens the input and output Ascii files, calls a configuration analysis routine, and closes the files after the analysis is completed. In the program, a common block array, Prop, serves as a property database. This array holds the component and transmission property values from the analysis subroutines. It is two dimensional, with columns corresponding to specific components of a transmission and rows containing values for specific properties. The first column of the whole array contains the system properties for the entire transmission, whereas the first column for each unit transmission contains the system properties for that unit.

The main program calls one of three configuration analysis routines to analyze each unit transmission. The configuration routines match the configuration groupings of Figs. 1-3.

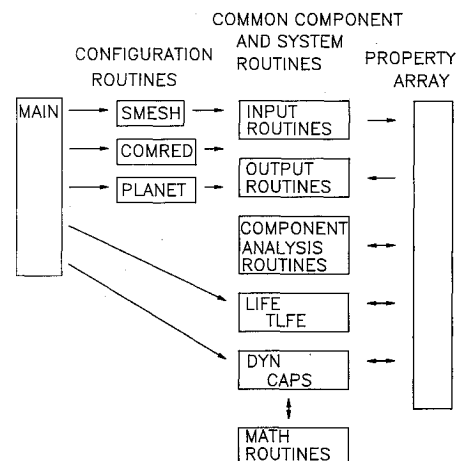


Fig. 5 Tlife program block diagram.

Smesh analyzes the single mesh unit transmissions, Comred analyzes the compound transmissions, and Planet analyzes the planetary and star configurations.

As indicated in Fig. 5, the program is modular in that the configuration-specific subroutines use the same input and output routines, component analysis routines, basic mathematics subroutines, and system analysis subroutines. For each unit transmission, one of the analysis routines reads in the input data, calculates the component properties, and performs component life and capacity analyses. The main program then calls the system life and capacity routines to perform the system analysis and uses the output routines to write the output file.

Since the subroutines that determine the transmission life and dynamic capacity interface directly with the property array, they are separated from any specific transmission configuration. The system analysis subroutines, Life and Dyn, work in an identical manner for all unit transmission configurations. For the full system, the same routines are used with a pointer array to include all of the components in the overall transmission while skipping the unit transmission properties. Thus, other configurations can be added to the program by adding appropriate configuration analysis subroutines.

Analysis Methods

For each configuration the analysis begins with a static and kinematic analysis of the reduction to determine the gear and bearing loads and load cycles. Dynamic capacities are then determined for the gears and bearings in the unit transmission. These capacities are used with the Lundberg-Palmgren life model to determine the service life of the transmission from the component loads and load cycles.

The life model comes from rolling element bearings.¹² Lundberg and Palmgren determined that the scatter in the life of a bearing can be modeled with a two-parameter Weibull distribution in terms of a 90% probability of survival life:

$$\ell n(1/R) = \ell n(1/0.9) \cdot (l/l_{10})^b \quad (1)$$

This life to reliability relationship is at a specific load F which corresponds to the l_{10} life. This load F is related to the component basic dynamic capacity C as

$$l_{10} = (C/F)^p \quad (2)$$

Since the life at the dynamic capacity is one-million load cycles, it does not appear as a variable in the equation.

These equations, which are based on experiment, give the life of the component for a given load F and reliability level R . By knowing the Weibull slope b , the load-life exponent p , and the dynamic capacity of the component C , one can use these equations to calculate the life of the component for any desired reliability.

The STLE life factors committee modified Eq. (2) with life and load adjustment factors.¹² These factors extend Eq. (2) to cover many different end-use situations so that designers can size bearings properly. The revised equation for $l_{10,a}$, which is used in this analysis, is

$$l_{10,a} = a(C/vF_r)^p \quad (3)$$

In Eq. (1), b is normally 10/9 for ball and 9/8 for straight roller bearings, and in Eqs. (2) and (3) p is 3.0 for ball bearings and 3.33 for roller bearings.

Since gear tooth pitting failures are similar to bearing failures, with the possible difference of surface initiation, the two-parameter Weibull distribution can be used to describe the scatter in gear life with a different Weibull slope. The load-life relationship of Eq. (2) is given graphically in the ANSI/AGMA standards as a life factor for gears.¹³

To combine these component life models into a model for the service frequency of a transmission, the drive system re-

liability R_s is treated as a strict series probability of all the component reliabilities.² This makes the system reliability the product of the reliabilities of all the components. The system life can also be expressed as a two-parameter Weibull distribution in terms of the system reliability parameters, b_s and $l_{10,s}$:

$$\ell n(1/R_s) = \ell n(1/0.9) \cdot (l_s/l_{10,s})^{b_s} \quad (4)$$

The system Weibull slope b_s and system 90% reliability $l_{10,s}$ can be found by iteration. The system dynamic capacity and load-life factor are found by a similar iteration.

This nominal, high-reliability life can be converted to an estimated service life by calculating the average time to failure for a large number of units in service. The mean time between overhauls for full transmission replacement is this system mean life, which is related to the 90% reliability life by the gamma function Γ

$$l_{av} = \frac{l_{10} \cdot \Gamma(1 + 1/b)}{[\ell n(1/0.9)]^{1/b}} \quad (5)$$

If the transmission repairs are component repairs rather than full replacement, then a second mean life is required to estimate the mean time between overhauls. This second mean life is the average mean life of the individual components. It is based on a different statistical model in which each component's failure rate is constant and independent. In this model, the transmission failure rate is the sum of the component failure rates. Using the reciprocal of the failure rate as an estimate for the mean life gives

$$l_{av,s} = 1 / \sum_{i=1}^n (1/l_{av,i}) \quad (6)$$

This second mean transmission life is labeled the mean component life in the output.

Applications

Two examples are presented to illustrate the program. The results of the Allison T56/501 turboprop reduction gearbox comparison⁶ are also discussed at the end of this section. Figure 6 is a schematic of the first example, a spiral bevel reduction that transmits 105 kW of power at an input speed of 4000 rpm and torque of 250 N-m to an output speed of 2000 rpm and torque of 500 N-m. The gears have 21 and 42 teeth, a back-cone distance of 120 mm, and a face width of 30 mm with a shaft angle of 110 deg. The bearings have 55-mm bores and are located 75 and 200 mm behind their respective gear centers. The bearings are 300 series with the close bearings being roller and the far bearings being ball and supporting the thrust loads.

Table 1 lists the equivalent radial loads, adjusted basic dynamic capacities, and mean lives of the four bearings, and two

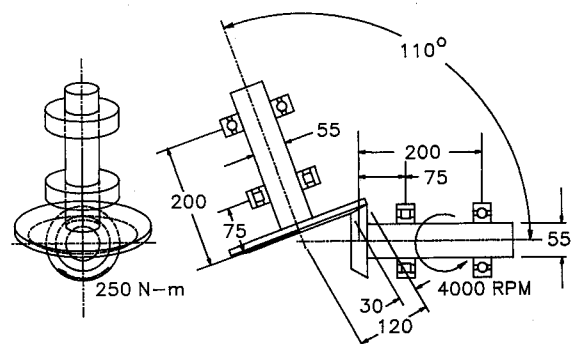


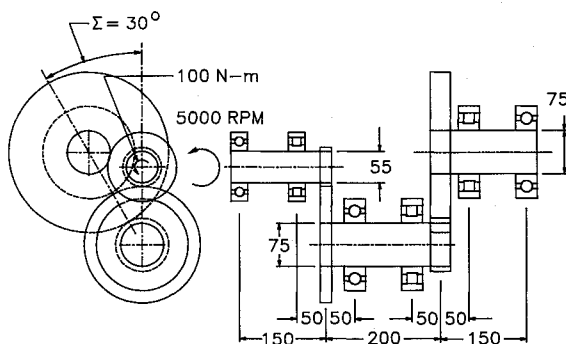
Fig. 6 Spiral bevel reduction.

Table 1 Spiral bevel reduction loads and lives

Element	Load, kN	Adjusted capacity, kN	Mean life, H
Pinion	6.3	34.1	8,240,000
Pinion roller bearing	7.8	52.3	3,430
Pinion ball bearing	4.8	31.9	1,940
Gear	6.3	35.7	12,490,000
Gear roller bearing	9.0	76.2	7,570
Gear ball bearing	3.0	47.5	25,655
Transmission	—	—	1,145
Components	—	—	1,020

Table 2 Two-stage helical reduction loads and lives

Element	Load, kN	Adjusted capacity, kN	Mean life, H
Input pinion	3.6	14.6	533,675
Input roller bearing	4.6	44.9	11,845
Input ball bearing	2.1	27.1	14,640
First gear	3.6	15.7	1,031,690
Intermediate roller bearing	12.8	135.6	15,820
Intermediate ball bearing	7.0	72.4	7,370
Second pinion	7.6	33.7	1,351,390
Output gear	7.6	36.3	2,612,485
Output roller bearing	12.6	259.1	139,640
Output ball bearing	6.4	142.9	75,530
First stage	—	—	7,040
Second stage	—	—	5,015
Transmission	—	—	3,145
Components	—	—	2,660

**Fig. 7 Sequential single helical reduction.**

gears and the life of the unit transmission as found by the program. The loads and capacities are in force units since they are both directly on the components. The adjusted capacity is the load that would produce a one-million load-cycle life with a 90% reliability for that component, including life and load adjustment factors. In this example one can see the strong influence of the bearing capacities on the transmission service life. One can also see that the service life is greater at 1145 hours for replacing the entire transmission than 1020 hours for replacing only the failed component in a maintenance session. By replacing the entire transmission, one replaces components with partial damage in addition to the failed component, extending the service life of the repaired transmission at a larger material cost per maintenance. These two lives are close to each other, due to the short life of the input shaft ball bearing relative to the lives of the other components in the transmission.

Figure 7 shows the second example, a two-stage helical reduction that takes an input torque of 100 N-m at 5000 rpm and converts it to an output torque of 900 N-m at 556 rpm at a power level of 52 kW. The transmission has four gears and six bearings and was modeled both as a compound helical

reduction and as two single-mesh helical reductions in series. Both analyses yielded the same results. The two gear reductions are each 3:1, with the first one having a 13-tooth pinion and a 39-tooth gear and the second one having a 14-tooth pinion and a 42-tooth gear. The input gear mesh has a normal module of 4.5 mm, a helix angle of 30 deg, and a face width of 20 mm. The output gear mesh has a module of 6 mm, a helix angle of 25 deg, and a face width of 35 mm. All bearings are 300 series, with the input bearings having a 55-mm bore and the other four bearings having a 75-mm bore. Each shaft has one ball bearing and one roller bearing, with the ball bearing carrying the lighter radial load and the thrust load. The near bearings are 50 mm from their respective gears and the far bearings are 150 mm from the gears as shown in Fig. 7.

Table 2 lists the equivalent radial loads, adjusted dynamic capacities, and mean lives of the six bearings and four gears, and the lives of the unit transmissions as found by the program. Once again, the bearings are the critical elements in determining the life of the transmission. The input shaft sees the highest load cycle count at the highest speed, but has the lowest loads and the smallest bearings. In contrast, the output shaft has the lowest speed and high loads with larger bearings. The intermediate shaft has the same size bearings as the output shaft, but has higher loads and a higher speed than the output shaft. This combination of loading, speed factor derating, and high cycle-count make these bearings significant in the life analysis. As in the first example, the transmission service life is shorter than that of the weakest component, but is close to it indicating that the design is somewhat out of balance and is service limited by a single component. With the larger number of components, there is a greater disparity between the full transmission replacement service life of 3145 h and the failed component replacement service life of 2660 h. Analyses such as the one available through this program can assist the designer in obtaining more optimal designs at an early stage.

To verify the method, a comparison analysis of the service life requirements for an Allison T56/501 turboprop reduction gearbox was performed.⁶ This gearbox has a two-stage reduction: a spur mesh followed by a five planet planetary with an overall reduction of approximately 13.54:1. Reported power levels are from 1–3 MW. Both the computational analysis and the field service data identified the planet bearings as requiring the most frequent service. The bearings on the output and input shafts of the spur gear mesh were identified as the next in frequency of service. To predict the actual field service replacement frequency, the analytical model required life service factors of 12 for the input shaft bearings and 6 for the other bearings. Although the unadjusted analytical predictions were more conservative than actual field service data, they did identify the critical components in the transmission from a service standpoint.

Conclusions

A generalized life and reliability model has been applied to predict the service life of aircraft transmissions composed of spur, helical, and spiral bevel gears with ball, straight roller, and tapered roller bearings. Models such as this are needed to enable the evaluation of transmission service life before the transmission is constructed. With this model, one can optimize the transmission to maximize the system service life while maintaining given power and weight constraints.

The model has been applied to single mesh transmissions as well as compound and planetary transmissions and transmissions composed of unit transmissions in series. The life model for this analysis is the two-parameter Weibull failure distribution, which has been applied to the bearings and gears of the transmissions as well as the overall transmission systems.

The program that performs these analyses has been described and its use illustrated with two design examples. A spiral bevel single mesh transmission was analyzed for its ser-

vance life at a given power level as specified by its input torque and speed. A compound helical gear transmission was analyzed first as a single compound reduction and again as two single mesh reductions in series to illustrate the flexibility of the program. This transmission was analyzed for its service life at a specified power level. The separate service lives of the two component parts of this transmission were presented as well. Both analyses of the compound helical gear transmission produced the same results.

Component analyses indicate the service life-limiting factors in the transmissions so they can be overcome to improve the overall life of the transmission before the transmission is constructed.

References

- ¹Ludemann, S. G., "Prop-Fan Powered Aircraft—An Overview," Society of Automotive Engineers, SAE Paper 820957, Aug. 1982.
- ²Savage, M., and Lewicki, D. G., "Transmission Overhaul and Replacement Predictions Using Weibull and Renewal Theory," *Journal of Propulsion and Power*, Vol. 7, No. 6, 1991, pp. 1049–1054.
- ³Dietrich, M., Stajszczak, M., and Szopa, T., "Computer Aided Gear Reliability Evaluation," *Proceedings of the International Conference on Motion and Power Transmission* (Hiroshima, Japan), Japan Society of Mechanical Engineers, Tokyo, Japan, 1991, pp. 502–507.
- ⁴Joachim, F. J., "Application of Modern Calculation Methods to Gears in Vehicle Transmissions," *Proceedings of the International Conference on Motion and Power Transmission* (Hiroshima, Japan), 1991, pp. 583–588.
- ⁵Coy, J. J., Townsend, D. P., and Zaretsky, E. V., "Dynamic Capacity and Surface Fatigue Life for Spur and Helical Gears," *Journal of Lubrication Technology*, Vol. 98, No. 2, 1976, pp. 267–276.
- ⁶Lewicki, D. G., Black, J. D., Savage, M., and Coy, J. J., "Fatigue Life Analysis of a Turbo-Prop Reduction Gearbox," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 108, No. 3, 1986, pp. 255–262.
- ⁷Savage, M., Paridon, C. A., and Coy, J. J., "Reliability Model for Planetary Gear Trains," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 105, No. 3, 1983, pp. 291–297.
- ⁸Savage, M., Brikmanis, C. K., Lewicki, D. G., and Coy, J. J., "Life and Reliability Modeling of Bevel Gear Reductions," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 110, No. 2, 1988, pp. 189–196.
- ⁹Savage, M., Radil, K. C., Lewicki, D. G., and Coy, J. J., "Computerized Life and Reliability Modeling for Turbo-Prop Transmissions," NASA TM 100918, July 1988; also *Journal of Propulsion and Power*, Vol. 5, No. 5, 1989, pp. 610–614.
- ¹⁰Savage, M., "Life and Dynamic Capacity Modelling for Aircraft Transmissions," NASA CR 4341, Jan. 1991.
- ¹¹Savage, M., Prasanna, M. G., and Rubadeux, K. L., "TLIFE—A Program for Spur, Helical and Spiral Bevel Transmission Life and Reliability Modeling," NASA CR 4622, Aug. 1994.
- ¹²Zaretsky, E. V., "STLE Life Factors for Rolling Bearings," Society of Tribology and Lubrication Engineers, Park Ridge, IL, 1992.
- ¹³"Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth," American Gear Manufacturers Association Standard, American National Standard Institute/American Gear Manufacturers Association 2001-B88, Alexandria, VA, Sept. 1988.